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RESEARCH AID

THEORY AND APPLICATIONS OF THE LEARNING CURVE

CIA/RR RA-7

24 July 1956

CENTRAL INTELLIGENCE AGENCY

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(ORR Project 33.978)

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FOREWORD

Important indications of the capabilities, vulnerabilities, and intentions of a foreign country may be obtained through analysis of that country's expenditures on key military end items. This type of analysis is considerably complicated by the decline in labor costs and in total production costs of military equipment over time. Many of these items involve repetitive major assembly tasks using large inputs of labor. Although the first units are produced over relatively long periods of time and at relatively high costs, the unit cost declines significantly as experience is gained.

The application of a single price over an extended output could result in misleading distortions. The trend of learning, on the other hand, tends to approximate the trend of output costs. The trend of learning may also aid in evaluating the significance of cost-saving announcements and in providing the basis for predicting the probable cost trends of programs which may be undertaken in the future.

Producers in the US and other Western countries have observed this downward trend and have ascertained that it follows what has come to be known as a learning, progress, experience, or improvement curve. Similar trends are probably experienced in the USSR.

This research aid describes the nature of several types of these curves and indicates how they may be derived and applied.

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THEORY AND APPLICATIONS OF THE LEARNING CURVE

I. Introduction.

The purpose of this research aid is to explain the basic theory of the learning curve and to familiarize analysts with the use of the learning curve as a tool in research.

The basic theory of the learning curve is simple. A man learns as he works. The more often he repeats an operation, the more efficient he becomes, and the less the time required to perform the operation. The reduction in the time required, however, becomes less with each successive operation. These facts have long been recognized, but until a decade ago the fact that the rate of improvement is sufficiently regular to be predictable was not known.

II. History and Development.

Dr. T.P. Wright developed the theory of the learning curve while he was employed by the Curtiss-Wright Corporation, Buffalo, New York. In February 1936, his findings were published in the Journal of Aeronautical Sciences in an article entitled "Factors Affecting the Cost of Airplanes."

By World War II the fact that the direct labor* input per airplane declined regularly as the cumulative number of airplanes produced increased had been recognized. More important in wartime was the fact that the unit cost also progressively declined so that more airplanes could be produced with the same labor force and facilities.

This reduction in direct labor input per airplane might have been called rising productivity except for the fact that each time a new airplane model was put into production the process repeated itself: that is, the direct labor required to produce the first unit of the new model reverted back to approximately the labor required to produce the first unit of the preceding model, and the learning process (assuming similar weight and function) had to begin over again. Because of this repetitive characteristic the phenomenon was called learning.**

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* The term direct labor, as used in this research aid, refers to the labor expended on the airframe manufacturing process which consists of machining, processing, fabricating, and assembling. Direct labor does not include engineering, design, supervision, administration, or maintenance.

** The learning curve is sometimes referred to as the improvement curve, the experience curve, or the progress curve.

Not long after the publication of Dr. Wright's article, several US aircraft companies established certain standards of learning, which they used as bases for predicting direct labor input and cost. The US armed services became interested in the theory of the learning curve and after World War II sponsored a statistical study of direct labor input by the Stanford Research Institute. The conclusions reached in this study were published in 1949.

By 1949 a series of learning curves, which represented the average experience in the production of various types of airframes such as fighters and bombers, had been developed. These curves all were different in terms of the direct labor input required to produce the first unit of a particular type, but they had one characteristic in common, the rate of improvement.

The average rate of improvement for all US aircraft companies was approximately 20 percent between doubled quantities of units produced. This rate of improvement was referred to as an 80-percent learning curve: that is, the direct labor required to produce the second unit was 80 percent of that required to build the first; for the fourth, 80 percent of that required to build the second; for the 400th, 80 percent of that required to build the 200th; and so forth.

It may seem unreasonable to expect this reduction in direct labor input to continue indefinitely because the labor required eventually would appear to reach an absurdly low figure. The quantity of units produced, however, must be doubled for every 20-percent reduction in the required labor input so that, although the labor required per unit steadily decreases, the cumulative output approaches infinity much faster than does the corresponding labor input. When large quantities of the same item have been produced, the rate of improvement in relation to the required production time may be so small as to seem to have reached a plateau where no further improvement is possible. Most US aircraft companies operate in the range between unit 1 and unit 1,500, and the rate of improvement therefore is perhaps most noticeable in the aircraft industry.

III. The Learning Curve on Arithmetic Graph Paper.

If an aircraft company operates on an 80-percent learning curve and is building a new model and if 100,000 man-hours are required to build the first unit, only 80,000 man-hours will be required to build the second unit. Production has been doubled, and only 80 percent as much time is required to build the second unit. If production is doubled again, only 64,000 man-hours will be required for the fourth unit. Thus the learning process continues. The following tabulation shows the direct labor required when the quantities of the theoretical airframe produced are doubled:

Unit Number	Direct Labor Required (Man-Hours)
1	100,000
2	80,000
4	64,000
8	51,200
16	40,960
32	32,768
64	26,214
128	20,972

Figure 1* shows the preceding tabulation plotted on arithmetic graph paper. On arithmetic graph paper the line is a "true curve" and dramatically shows the reduction in direct labor required as succeeding airframe units are produced. The line dips sharply at first and then begins to slope downward more gently as the percentage rate of improvement is spread over an increasingly larger volume of production.

Although the percentage rate of improvement is constant, a constant rate is difficult to interpret on arithmetic graph paper. As the number of units produced increases in geometric progression, the variable (time, price, or the like) decreases in geometric progression. To interpret the curve, therefore, knowledge of analytical geometry is necessary. Consequently, arithmetic graph paper usually is not used to show the learning curve. Another disadvantage in the use of arithmetic paper is the difficulty in showing unit 1 and unit 1,000 on the same graph without making the graph impractically large. For these and other reasons, the learning curve usually is plotted on log-log** graph paper.

IV. The Learning Curve on Log-Log Graph Paper.

Figure 2*** shows the same information as is shown in Figure 1 plotted on log-log graph paper. When the learning curve is plotted on log-log graph paper, it follows a straight line and therefore is easy to interpret and easy to project.

The curve in Figure 2 may appear to be too steep, but it is not. Because of the expanding scales of log-log graph paper, the curve actually decreases at a decreasing rate and therefore the number of man-hours approaches zero at infinity.

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* Following p. 4.

** When the word log appears as an adjective, it is an abbreviation for logarithmic; when it appears as a noun, it is an abbreviation for logarithm.

*** Following p. 4.

The main difference between arithmetic graph paper and log-log graph paper is that log-log graph paper is laid out so that the distance between doubled quantities is equal. Because of this fact, the learning curve on log-log graph paper follows a straight line. If the percentage rate of decrease between doubled quantities is equal and if the distance on the graph between doubled quantities is equal, the line will be straight. The learning curve can be plotted easily with a ruler on log-log graph paper.

V. Learning Curves of Various Percentages.

Each US aircraft company operates differently. Each company therefore shows learning at different percentage rates. The steepness of the learning curve depends upon the percentage rate of learning. Some companies show a rate of learning of 70 or 75 percent, whereas other companies show a rate of learning of 85 or 90 percent. Figure 3* shows some of the different percentage rates of learning plotted on log-log graph paper.

The percentage rates of learning of selected US aircraft companies during World War II are shown in Table 1.** Based on these rates of learning, the slopes of the learning curves for direct labor input in series production of airframes generally range from 73 to 88 percent. The degree of slope of the learning curve is influenced by a number of factors, such as the following: job familiarization (supervisors and workmen), tool coordination, shop organization (lack of balance, job assignments, congestion, coordination between shops, shop load, and schedule status of other shops), engineering liaison, jig alignment, parts supply (availability and quality of work previously done on parts by other shops), handling time by cranes and the like, inspection (official and other), time to change jobs (check-in procedure), personal factors (fatigue, morale), modifications and changes, turnover of personnel, modernization of tooling, shifting work into subassembly, small tools (issuance, quality), tolerances, shift changes, bench and hand work, types of construction or processing, suggestion systems, specialization of operations, schedule increases, and quantities produced.

Although the slopes of the learning curves for direct labor input in series production of airframes generally range from 73 to 88 percent, the slope trends vary somewhat for different operations. Learning curves for the production of detail parts are several degrees flatter than the average curve for the production of the whole airframe; curves for the production of subassemblies usually are compatible

* Following p. 4.
** Table 1 follows on p. 5.

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FIGURE 1

THE 80-PERCENT LEARNING CURVE PLOTTED ON ARITHMETIC GRAPH PAPER

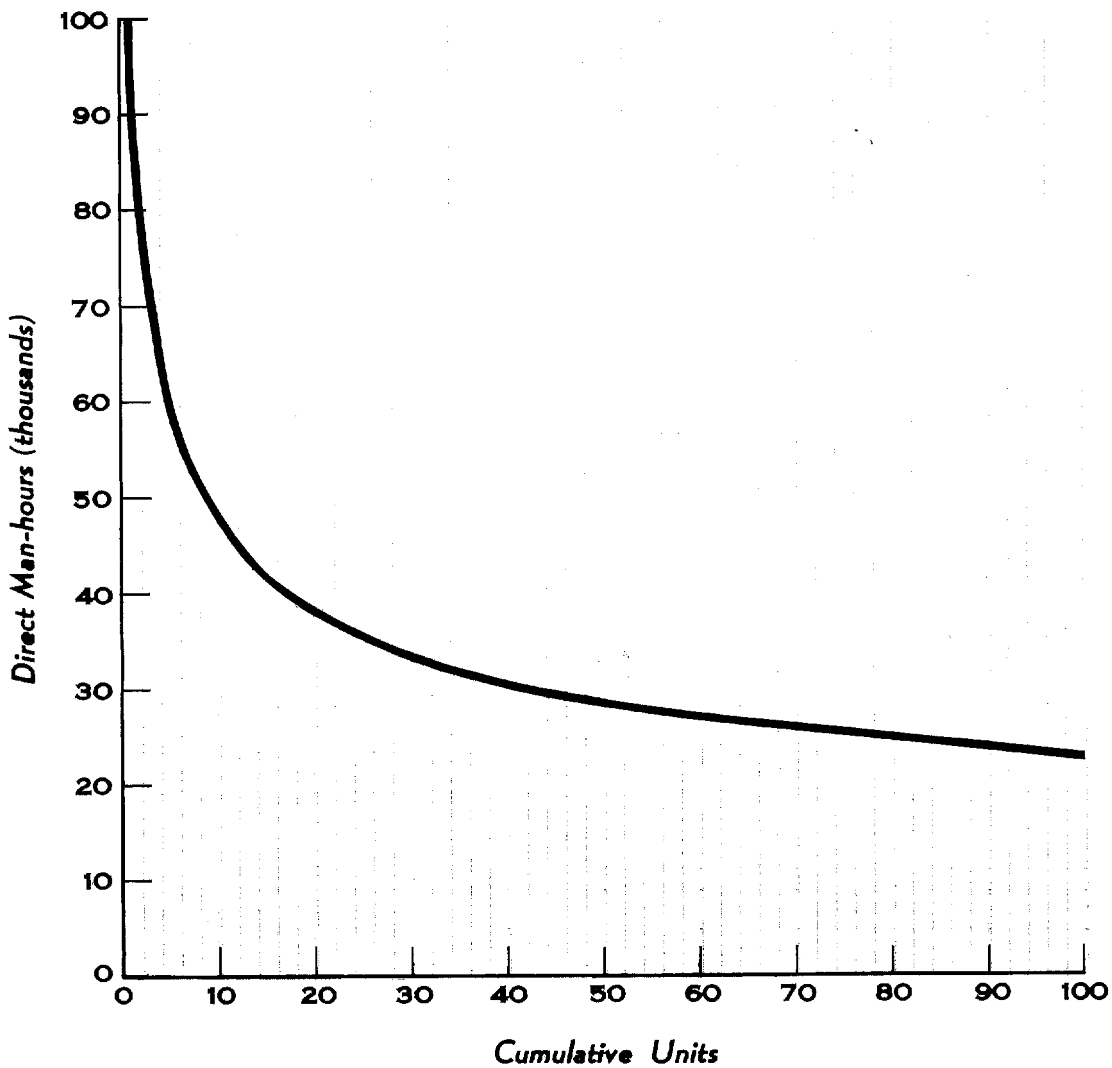

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FIGURE 2

THE 80-PERCENT LEARNING CURVE PLOTTED ON LOG-LOG GRAPH PAPER

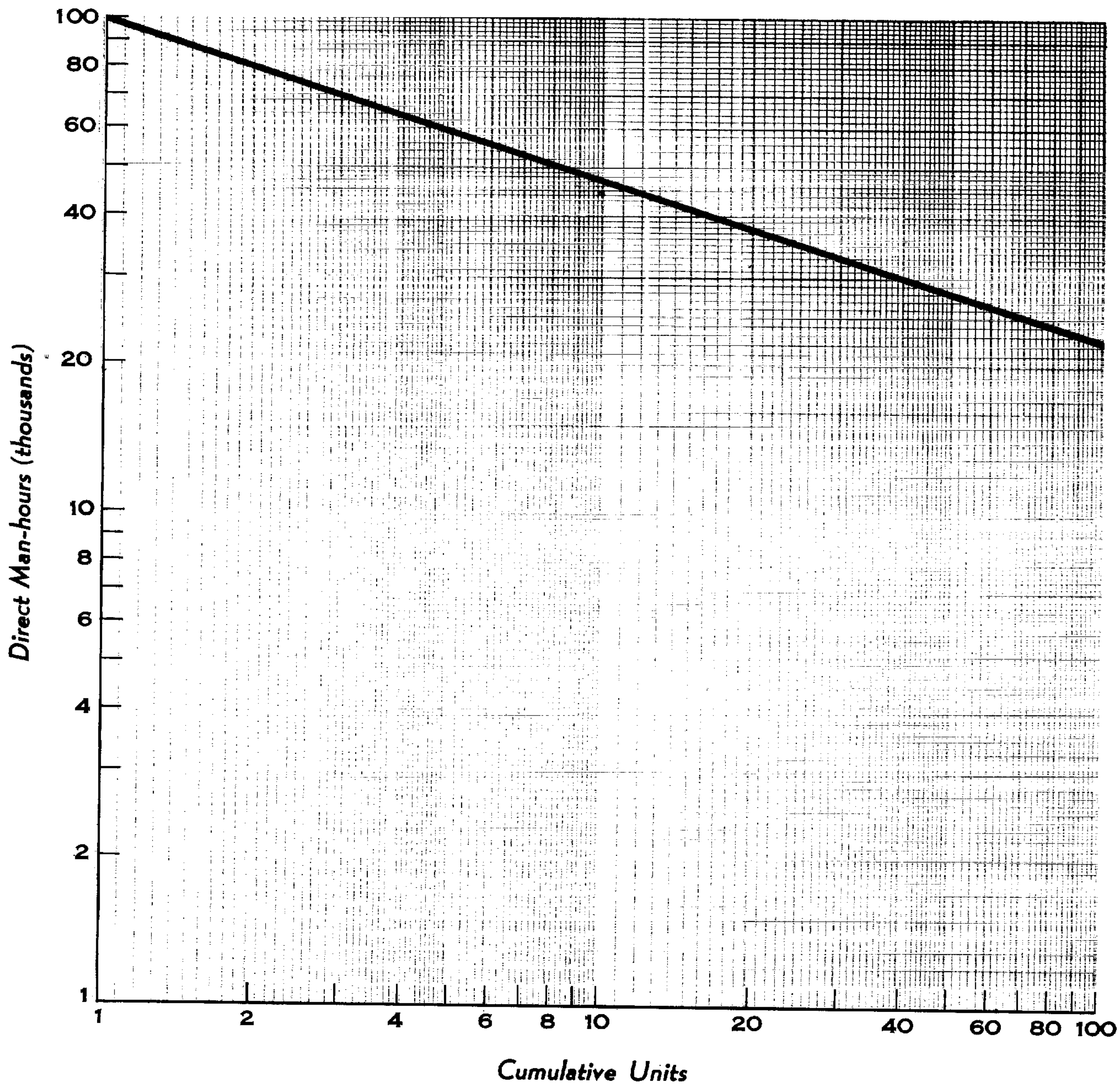

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FIGURE 3

LEARNING CURVES OF VARIOUS PERCENTAGES

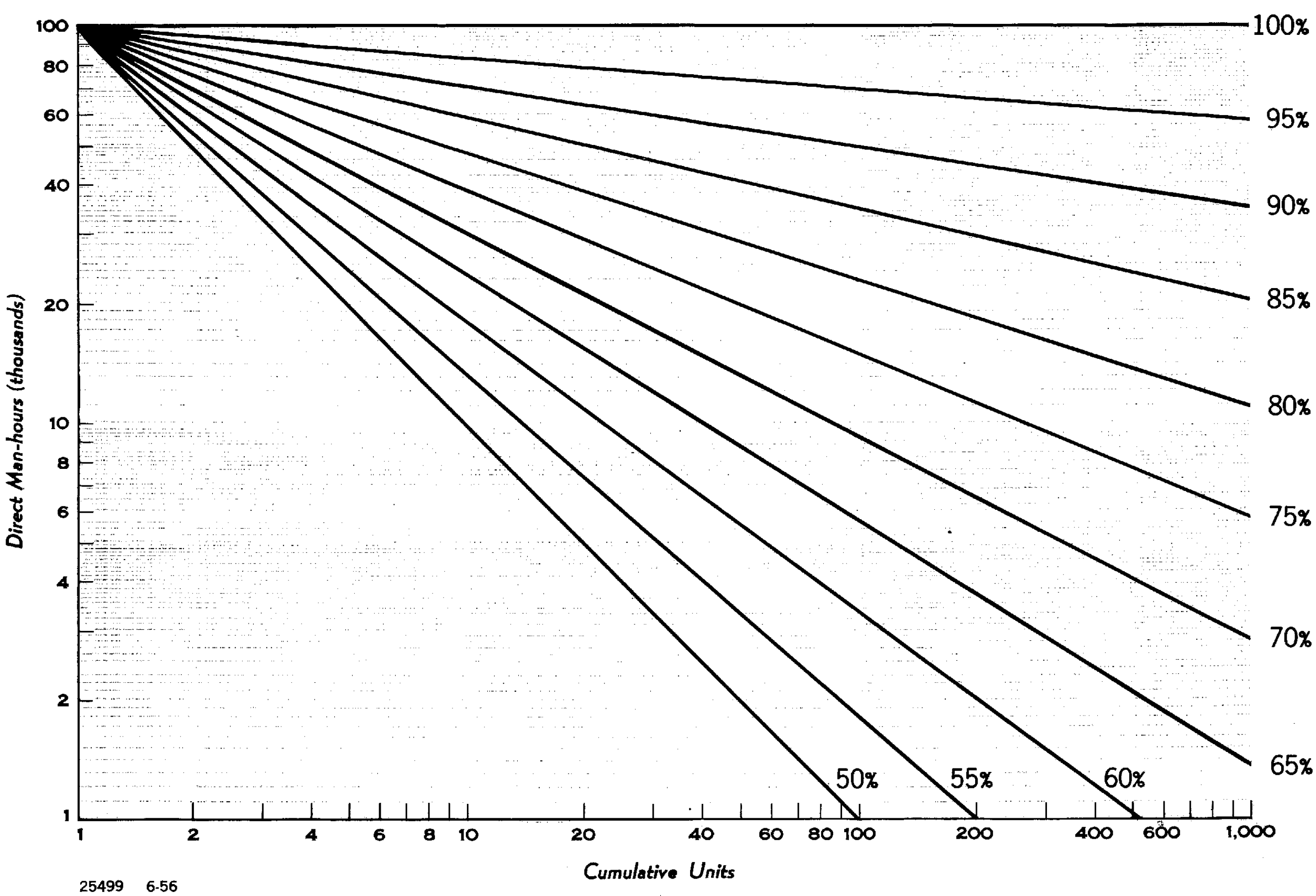

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Table 1

Percentage Rates of Learning of Selected US Aircraft Companies During World War II a/

Company	Aircraft Model	Percentage Rates of Learning
Beech Aircraft Corporation, Wichita, Kansas	AT-10	76.7
Boeing Airplane Company, Renton, Washington	First 400 B-29's	80.5
Boeing Airplane Company, Renton, Washington	Last 700 B-29's	79.0
Boeing Airplane Company, Wichita, Kansas	First 900 B-29's	71.8
Boeing Airplane Company, Wichita, Kansas	Last 800 B-29's	69.5
Convair Consolidated Vultee Aircraft Corporation, Fort Worth, Texas	First 1,000 B-24D's	76.4
Douglas Aircraft Company, Incorporated, Long Beach, California	First 1,000 B-17's	77.4
Douglas Aircraft Company, Incorporated, Tulsa, Oklahoma	B-24 E	75.0
Ford Motor Company, Willow Run, Aircraft Division, Ypsilanti, Michigan	B-24	70.8
Lockheed Aircraft Corporation, Burbank, California	B-17	65.3
North American Aviation, Incorporated, Dallas, Texas	B-24	75.0
North American Aviation, Incorporated, Dallas, Texas	AT-6	98.0
Republic Aviation Corporation, Farmingdale, New York	P-47 N	89.0

a. Army Air Forces, Materiel Command, Wright Field, Dayton, Ohio, and Industrial Mobilization Office. Source Book of World War II Basic Data, Airframe Industry, vol 1, Direct Man-Hours -- Progress Curves, April 1952. U.

with the average curve for the whole airframe; and curves for final assembly operations are several degrees steeper than the average curve for the whole airframe. The slope of the detail parts curve is flatter, for example, because of the influence of machine operations. Machines have a definite limit of output. The only improvement which can be expected in machine operations is a reduction in set-up time and in handling time.

The range of slopes in the selling price curves which result from the learning curves noted above is not so wide as in the corresponding learning curves, because all other cost elements (with the exception of production overhead applied to labor) temper the effect of labor costs on the total selling price. The reduction in material costs per unit as a result of series production, for example, is insignificant as compared with the reduction in direct labor required. The slope of the selling price curves for airframes produced in series generally ranges from 88 to 95 percent.

VI. Mathematical Development.*

The mathematical development of the learning curve is not difficult. The use of correlation and other statistical methods has shown that a graph of actual performance data may be described by an equation of the following type:

$$y = \frac{K}{x^n}$$

This equation** is the theoretical learning curve and may be used to describe the learning curve unit time curve, cumulative average time curve, and total time curve. Although the equation will describe actual data, the mathematical exactness of the theoretical learning curve will not permit these three curves all to be straight lines simultaneously on log-log graph paper. Learning curves may be considered in the following three classes:

1. Where the cumulative average time curve and the total time curve are of the type

$$y = \frac{K}{x^n}$$

* For problems based on the learning curve, see Appendixes A and B.

** K = a constant = value of y when x = 1.
n = the tangent of the angle which the straight line curve forms with the x axis.

2. Where only the unit time curve is of the type

$$y = \frac{K}{x^n}$$

3. Where all three time curves are modifications of classes 1 and 2, above.

A. Class 1.

1. Cumulative Average Time Curve.

The cumulative average time curve, shown in Figure 4,* has been developed by the equation

$$y = \frac{K}{x^n} \tag{1}$$

where

y = cumulative average time in direct man-hours required to produce any number of units,
K = number of direct man-hours required to produce the first unit,
x = any number of units produced, and
n = slope (tangent) of the learning curve.

2. Unit Time Curve.

The unit time curve parallels the cumulative average time curve shown in Figure 4, from unit 10 on. The unit cost in direct man-hours can be read along the vertical axis from the unit time curve. (The two curves meet at unit 1 because the unit time and cumulative average time are the same at that point.) The unit time curve gradually approaches a straight line that is parallel to, and somewhat lower than, the cumulative average time curve. This straight line is called the asymptote, and the unit time curve is said to be asymptotic to it. The values of all points on the asymptote are equal to $(1 - n)$ times the values of the respective points on the cumulative average time curve. The individual unit time for any unit x is approximately equal to the time shown on the asymptote at unit $x - 1/2$. Thus the unit time for unit 2 is equal to the value of the asymptote at unit number 1 1/2. This method may be used to approximate the values of the unit time curve through unit 10. For practical purposes, the unit time curve for units 10 and above may be considered as equal to the values of the asymptote.

* Following p. 8.

The approximate time for a single unit x greater than 10 is expressed by the following equation:

$$y = \frac{K}{x^n} (1 - n) \tag{2}$$

where y = direct man-hours required to produce a specific unit and all other symbols are the same as in equation (1). A closer approximation than equation (2) is

$$y = \frac{K}{(x - 1/2)^n} (1 - n) \tag{3}$$

3. Total Time Curve.

The total time curve in Figure 3* is expressed by the equation

$$y = Kx^{(1 - n)} \tag{4}$$

where y = total direct man-hours required to produce a specific number of units and all other symbols are the same as in equation (1).

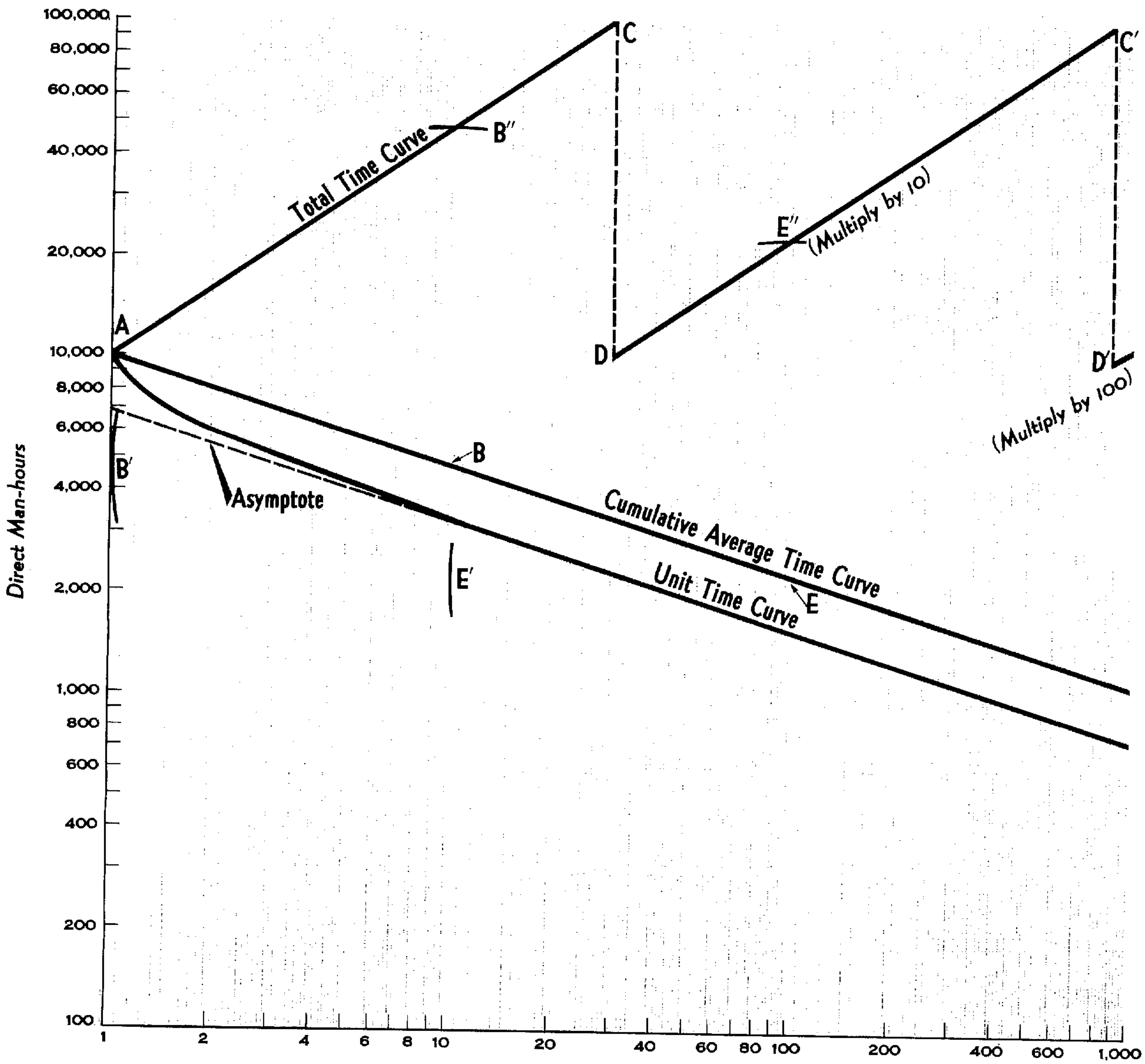
To find the total time curve take any point on the cumulative average time curve -- point B on Figure 4 -- and measure the distance horizontally back to unit 1. Erect this distance vertically over point B to produce a point B". Then connect the 2 points from the direct man-hours for unit 1 -- point A and point B" -- and extend. From this curve the total number of direct man-hours required to produce any number of units may be read. This procedure actually adds the logarithmic distance of the cumulative average time to the logarithmic distance of the units involved, thus multiplying the values.

Because the curve runs off the graph, take point D, vertically under point C and at the unit 1 level, and draw a new curve parallel to the one just drawn, which should be read in units 10 times as great. If parallel rulers are not available, choose another point E, measure back to E' at unit 10, and repeat the process.

* Following p. 4, above.

FIGURE 4

CUMULATIVE AVERAGE TIME, TOTAL TIME, AND UNIT TIME CURVES—CLASS I



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B. Class 2.

1. Unit Time Curve.

The unit time curve shown in Figure 5* has been developed by the equation

$$y = \frac{K}{x^n} \tag{5}$$

where

y = direct man-hours required to produce unit x,
K = number of direct man-hours required to produce the first unit,
x = any number of units produced, and
n = slope (tangent) of the learning curve.

2. Total Time Curve.

The summation of a series of unit man-hours may be closely approximated by using the definite integral from the first unit minus one-half to the last unit plus one-half. Thus the integration will be between the value one-half and the last unit number plus one-half, as follows:

$$\sum y \cong \int_{1/2}^{x + 1/2} Kx^{-n}\,dx = K \int_{1/2}^{x + 1} x^{-n}\,dx$$

$$\sum y \cong \frac{K}{1 - n} \left[(x + 1/2)^{1 - n} - (1/2)^{1 - n} \right] \tag{6}$$

where y = total direct man-hours required to produce all units through unit x and all other symbols are the same as in equation (5).

The total time curve shown in Figure 5 has been developed from equation (6).

* Following p. 10.

3. Cumulative Average Time Curve.

The cumulative average time curve in Figure 5 was developed from the equation

$$y = \frac{\frac{K}{1-n}\left[(x+1/2)^{1-n} - (1/2)^{1-n}\right]}{x} \tag{7}$$

where y = cumulative average time in direct man-hours required to produce each unit and all other symbols are the same as in equation (6).

The cumulative average time curve gradually approaches a straight line which is parallel to and somewhat higher than the unit time curve. The values of all points on the asymptote are equal to the values of the respective points on the unit time curve divided by $(1 - n)$.

C. Class 3.

1. Unit Time Curve.

The unit time curve shown in Figure 6* has been developed by research at Stanford University and may be determined by the following equation

$$y = \frac{a}{(x + B)^n} \tag{8}$$

where

y = direct man-hours required to produce unit x,
x = unit number of any unit,
a = constant expressed in terms of direct man-hours,
B = constant expressed in terms of units, and
n = slope (tangent) of the asymptote of the learning curve.

* Following p. 10.

FIGURE 5

CUMULATIVE AVERAGE TIME, TOTAL TIME, AND UNIT TIME CURVES—CLASS 2

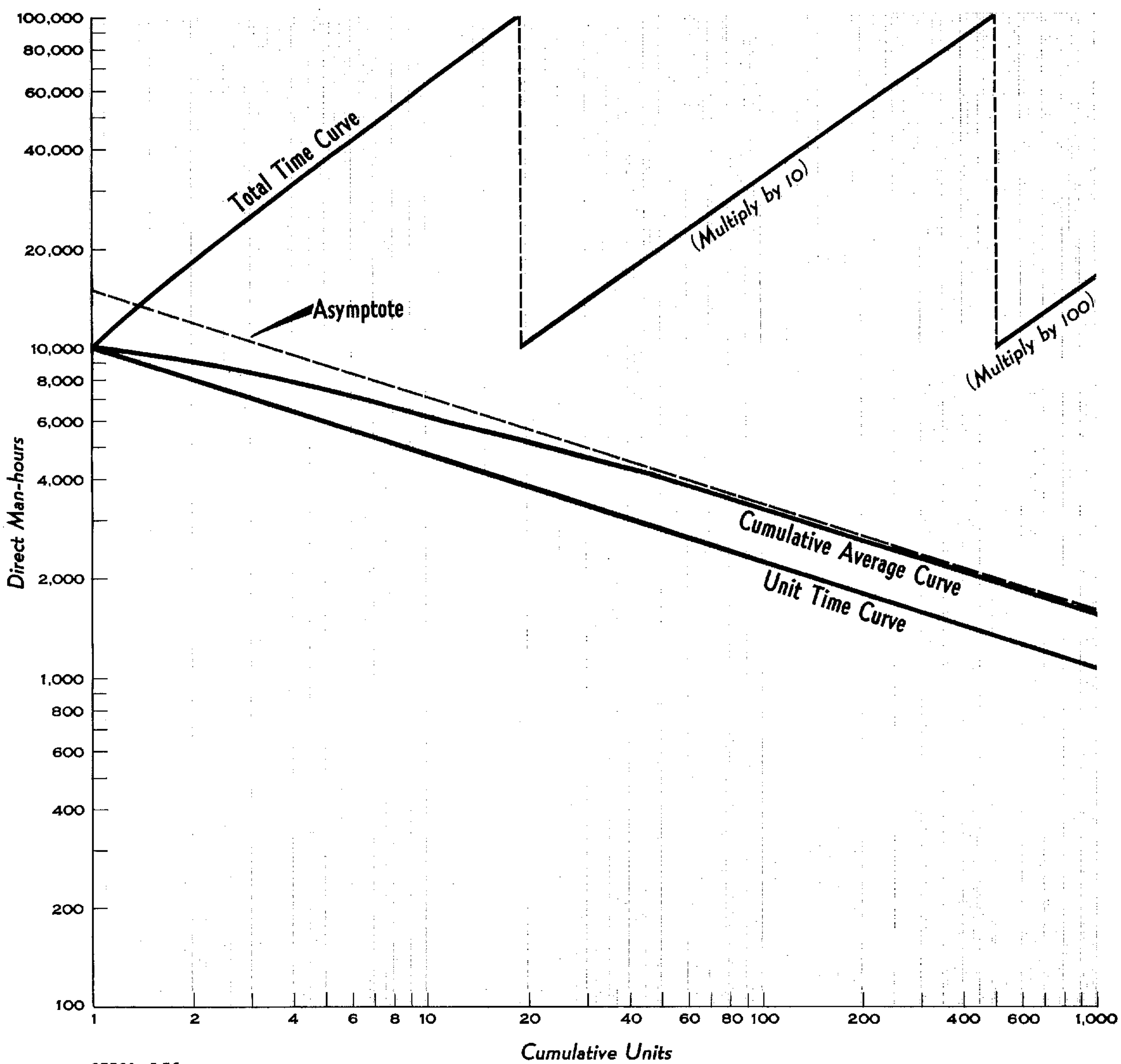

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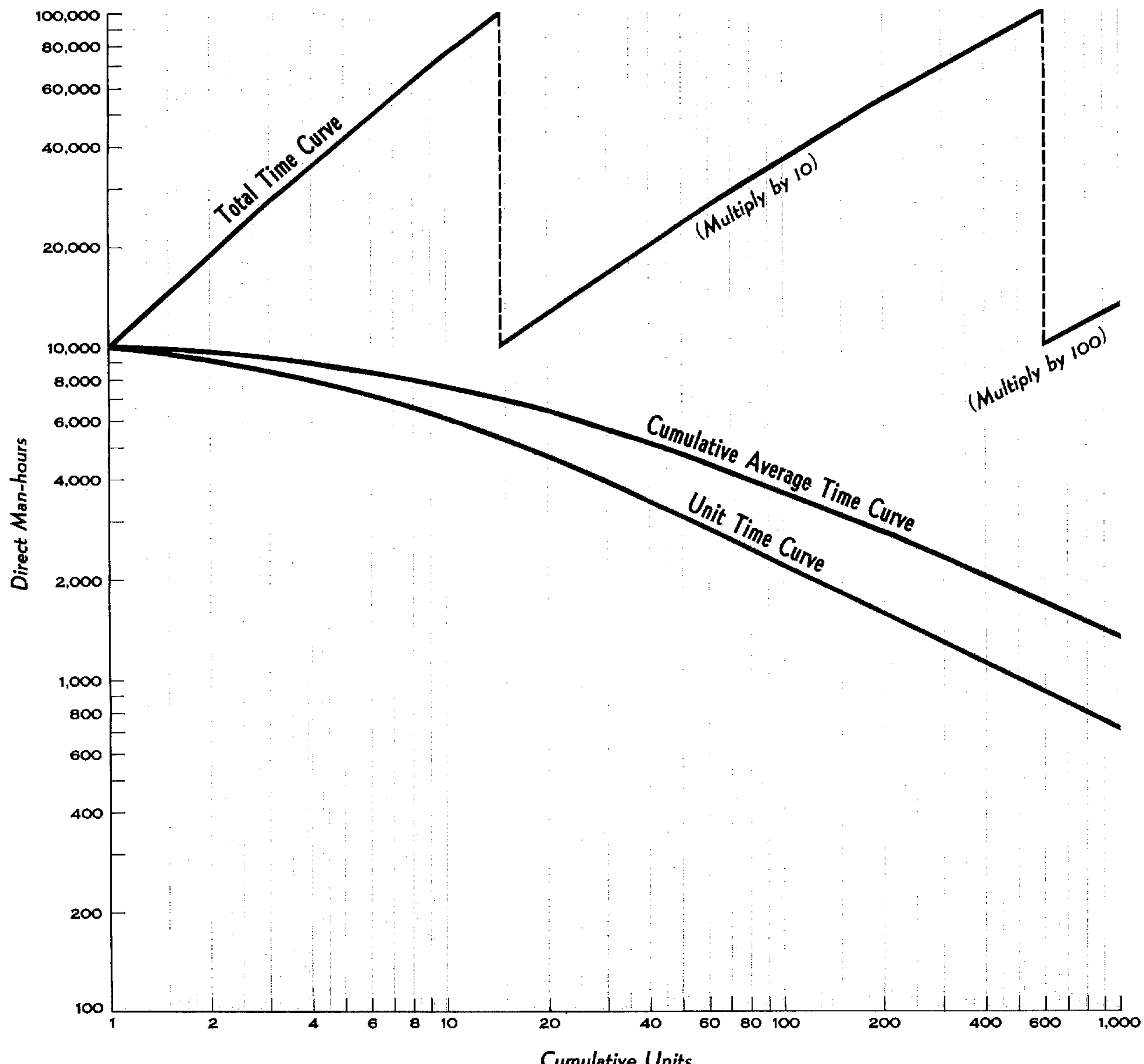
FIGURE 6

CUMULATIVE AVERAGE TIME, TOTAL TIME, AND UNIT TIME CURVES—CLASS 3



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2. Total Time Curve.

The total time curve shown in Figure 6* was developed by integrating equation (8) from the first unit minus one-half to the last unit plus one-half, as follows:

$$\sum y \cong \int_{1/2}^{x + 1/2} a\,(x + B)^{-n}\,dx \cong a \int_{1/2}^{x + 1/2} (x + B)^{-n}\,dx$$

$$\sum y \cong \frac{a}{1 - n} \left[(x + B + 1/2)^{1 - n} - (B + 1/2)^{1 - n} \right] \qquad (9)$$

where y = cumulative total direct man-hours required to produce all units through unit x and all other symbols are the same as in equation (8).

3. Cumulative Average Time Curve.

The cumulative average time curve shown in Figure 6 was developed from the equation

$$y = \frac{\frac{a}{1 - n} \left[(x + B + 1/2)^{1 - n} - (B + 1/2)^{1 - n} \right]}{x} \qquad (10)$$

where y = cumulative average direct man-hours required to produce each unit and all other symbols are the same as in equation (8).

VII. Determination of the Slope of a Theoretical Learning Curve.

Equation (1) can be solved for the slope of a learning curve by taking the log of both sides of the equation, which gives

$$\log y = \log K - n \log x \qquad (11)$$

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* Following p. 10, above.

To find the numerical value of slope n for an 80-percent learning curve, proceed as follows:

> By definition, the percent of the learning curve is the ratio of the man-hours required to produce unit 2 to the man-hours required to produce unit 1. Let K = 100 and y = 80, where x = 2.

In equation (5)

$$\text{Log } 80 = \log 100 - n \log 2$$

$$1.90309 = 2.00000 - 0.30103\, n$$

$$n = \frac{2.00000 - 1.90309}{0.30103} = \frac{0.09691}{0.30103} = 0.32193$$

Table 2* shows the numerical value of the slope n for various learning curves from 50 to 100 percent.

VIII. Determination of the Slope of an Actual Learning Curve.

The simplest method of determining the slope of an actual learning curve is to plot the data on log-log graph paper and draw a straight line as nearly through all the points as possible. The ratio of unit 2 to unit 1 indicates the percentage of the curve. From Table 2* the slope may be read.

Another method of determining the slope of an actual learning curve is to solve simultaneously the following equations, which are a modification of equation (2):

$$\sum n \log x + \sum \log y = \sum \log K$$

$$\sum n \log x_1 + \sum \log y_1 = \sum \log K$$

* Table 2 follows on p. 13.

Table 2

Numerical Value of the Slopes
of 50- to 100-Percent Learning Curves

Percent	Slope n	Percent	Slope n
50	1.0000	76	0.3959
51	0.9714	77	0.3771
52	0.9434	78	0.3585
53	0.9159	79	0.3401
54	0.8890	80	0.3219
55	0.8625	81	0.3040
56	0.8365	82	0.2863
57	0.8110	83	0.2688
58	0.7859	84	0.2515
59	0.7612	85	0.2345
60	0.7370	86	0.2176
61	0.7131	87	0.2009
62	0.6897	88	0.1844
63	0.6666	89	0.1681
64	0.6439	90	0.1520
65	0.6215	91	0.1361
66	0.5996	92	0.1203
67	0.5778	93	0.1047
68	0.5564	94	0.0893
69	0.5353	95	0.0740
70	0.5146	96	0.0589
71	0.4941	97	0.0439
72	0.4739	98	0.0291
73	0.4540	99	0.0145
74	0.4344	100	0.0000
75	0.4150		

This method is illustrated by an example in the following tabulation:

Unit Number	Direct Labor Required (Man-Hours)
1	204,131
2	176,747
3	152,546
4	144,123
5	139,713
6	131,538
7	128,221
8	113,812
9	114,407
10	109,624
11	103,323
12	102,618

The logs of x and y may then be determined and the equations solved, as follows:

x	y	Log x	Log y
1	204,131	0.00000	5.30991
2	176,747	0.30103	5.24736
3	152,546	0.47712	5.18341
4	144,123	0.60206	5.15872
5	139,713	0.69897	5.14523
6	131,538	0.77815	5.11906
		$\sum \log x = \underline{2.85733}$	$\sum \log y = \underline{31.16369}$
7	128,221	0.84509	5.10796
8	113,812	0.90309	5.05618
9	114,407	0.95424	5.05846
10	109,624	1.00000	5.03991
11	103,323	1.04139	5.01420
12	102,618	1.07918	5.01122
		$\sum \log x_1 = \underline{5.82299}$	$\sum \log y_1 = \underline{30.28793}$

$$2.85733\ n + 31.16369 = 6 \log c$$

$$\underline{5.82299\ n + 30.28793 = 6 \log c}$$

$$2.96566\ n - 0.87576 = 0$$

$$n = 0.29530$$

The resultant slope may be used to determine percent, as follows:

$$\log y = \log 100 - 0.29530 \log 2$$

$$\log y = 2.00000 - 0.08889$$

$$\log y = 1.91111$$

$$y = 81.5 \text{ percent}$$

IX. Uses of the Learning Curve.

The importance of the application of the learning curve may be seen in the following uses:

1. US aircraft companies use the learning curve to estimate costs when bidding for new contracts.

2. The US Government uses the learning curve to check the aircraft companies' bids for accuracy and reasonableness based on statistical analysis of the companies' records with respect to general production performance.

3. US aircraft companies use the learning curve to develop labor requirements, to estimate space and equipment requirements, to prepare budgets, to measure shop efficiency, and to check the progress of current contracts.

4. The US Government uses the learning curve to measure the efficiency and production dependability of aircraft companies.

5. US military planners use the learning curve to estimate aircraft mobilization expansion potential of the US.

X. Applicability.

Although the learning curve was developed and is used principally by the US aircraft industry, many types of industries should be able to apply the learning curve profitably. The usefulness of the learning curve depends upon the following factors:

1. Product Innovation.

Learning is an important factor in the performance of the workers in industries where major and minor design changes are frequent, where new products often are introduced, or where production is characterized by short production runs at well-separated intervals. These industries are near the top of their learning curves, where savings between units of production are significant much of the time.

2. Assembly Time as a Proportion of Total Time.

The more an operation consists of machine time as opposed to assembly time, the slower is the reduction in direct labor required. The US aircraft industry has found that there is a 20-percent or greater reduction in the time required for direct labor between doubled quantities in assembly operations but only a 10-percent reduction in the time required for direct labor machine-tool operations.

3. Advance Planning.

The more an operation can be planned in advance, particularly in terms of methods analysis and tooling, the more predictable will be the rate of reduction in the time required for direct labor. Learning in the literal sense tends to produce a smooth curve. Changes in methods, toolings, and the like during a production run make the learning curve uneven and probably give it a more pronounced slope.

Product innovation is extremely important in the electronics industry because of the long, complicated assembly lines in this industry. The electronics industry therefore should be able to make good use of the learning curve because the same conditions exist in this industry that have made learning curves helpful in the aircraft industry.

The learning curve may also be applicable to the shipbuilding industry. Direct labor represents a large percentage of the total costs of ships. Statistics on the direct labor costs of basically similar bulkheads and panels could be projected to cover units contemplated for production.

APPENDIX A

SOLUTION OF PROBLEMS IN WHICH THE CUMULATIVE AVERAGE TIME CURVE IS A STRAIGHT LINE ON LOG-LOG GRAPH PAPER OF THE TYPE $y = \frac{K}{x^n}$

I. Problem Number One.

A. Problem.

Experience indicates that an average time of 2,100 direct man-hours per unit is required to produce 25 hydraulic pumps. Experience also indicates an 84-percent learning curve for this type of production. How many direct man-hours would be required to produce 75 additional units?

B. Solution.

The computations would be as follows:

1. Direct man-hours required to produce unit 1 = $(y = \frac{K}{x^n})$

Then, $K = yx^n = 2{,}100\ (25)^{0.2515} = 4{,}715$ direct man-hours.

2. Cumulative average direct man-hours required to produce unit 100 = $y = \frac{4{,}715}{100^{0.2515}} = 1{,}871$ direct man-hours.

3. Direct man-hours required to produce 75 additional units = $(100 \times 1{,}871) - (25 \times 2{,}100) = 95{,}600$ direct man-hours.

II. Problem Number Two.

A. Problem.

A subcontractor had produced an initial lot of 130 fin assemblies at an average expenditure of 750 direct man-hours per fin assembly. At the completion of this lot, experience indicated that these fin assemblies were produced on an 82-percent learning curve. One year later the subcontractor received an order for 165 additional fin assemblies. How many direct man-hours would be required to produce 165 additional fin assemblies?

B. Solution.

Because production was disrupted for 1 year, much of the learning gained on the previous contract disappeared. It is therefore impossible to add 165 additional fin assemblies to the end of the initial 130 fin assemblies for projection purposes.

Experience has shown that the degree of learning "lost" can be measured to some extent. If a repeat order immediately follows an initial order and prevents disassembly of the production stations, additional units can be added immediately to the last unit of the initial order. If the production stations were to be disassembled for 4 to 8 months and then to be re-established to fill a repeat order, the advantage, or credit, gained from the learning supplied by the initial order would represent perhaps one-half of the initial learning. The repeat order of 165 fin assemblies, for example, would be subsequent to 65 units previously produced. If production were to lapse for about 1 year, the credit gained from the initial learning would be only one-quarter of the initial learning. Based on an initial order of 130 fin assemblies and a repeat order of 165 units 1 year later, the computations to determine the number of direct man-hours required to produce the 165 additional units would be as follows:

1. Direct man-hours required to produce unit 1 = $750 \times 130^{0.2863}$ = 3,020 direct man-hours.

2. Cumulative average direct man-hours required to produce unit 33 (25 percent of the initial order of 130 units = 33 units) = $\frac{3,020}{33^{0.2863}}$ = 1,110 direct man-hours.

3. Cumulative average direct man-hours required to produce 165 units after learning had been gained on 33 units = $\frac{3,020}{198^{0.2863}}$ = 665 direct man-hours.

4. Direct man-hours required to produce 165 additional units = 198 x 665 - 33 x 1,110 = 95,000 direct man-hours.

III. Problem Number Three.

A. Problem.

Twenty-five special-type cylinders involving machining and assembly operations have previously been purchased from a vendor at a price of US $250* per cylinder. The prime contractor plans to purchase 200 more cylinders. What would be a reasonable price for the repeat order?

* All dollar values are given in US dollars throughout this research aid.

B. Solution.

Because the prime contractor does not know the actual cost of the materials involved in the assembly of the cylinders, he would apply a price curve of 88 to 90 percent to the initial order. If no further minor changes are involved and if the repeat order is exactly the same as the initial order, an 88-percent price curve would be appropriate. If minor modifications are involved, a 90-percent price curve would be used. Assume that the former condition is applicable and that an 88-percent curve is used. The calculation would be as follows:

1. Cost of unit 1 = $250 \times 25^{0.1844} = \$452.65$

2. Average cost per unit for 225 units = $\dfrac{452.30}{225^{0.1844}} = \$166.71$

3. Total cost of the 200 additional units = $225 \times 166.71 - 25 \times 250 = \$31,259.75$

4. Average cost per unit for the 200 additional units $= \dfrac{31,259.75}{200} = \$156.30$

Vendors, in attempting to maintain a position in price on a repeat order often advance problems which are justifiable in many instances. These problems usually include (1) anticipated increases in labor and material costs, (2) overtime required on the repeat order, and (3) necessary changes. If the vendor's cost breakdown is not available because of the fixed price status or competitive nature of his quotation, but if his reasons are valid, consideration is given to the use of an 89- or 90-percent price curve.

IV. Problem Number Four.

A. Problem.

Fifty sets (1 right and 1 left) of elevator assemblies have been purchased from a subcontractor for $1,800 per set. It is planned to purchase 300 additional elevator sets to follow the initial 50 sets, with no appreciable gap in production. The subcontractor, however, has completed only one-half of the initial order of 50 sets and believes that the quotation of $1,800 will realize slightly less profit than originally anticipated. What should be the prime contractor's procedure to determine a realistic price for 300 additional elevator sets?

B. Solution.

A study of the subcontractor's original quotation and subsequent thinking in regard to the initial 50 elevator sets and a review of his position for the production of 300 additional sets disclosed the following general information:

1. $12,000 was estimated as the cost of tooling the first 50 sets. This amount was amortized over 50 sets by the subcontractor.

2. Estimated materials for the initial 50 sets cost $120.00 per set. Indications are that material costs will increase by approximately 4 percent during the period in which it is planned to procure materials for the 300 additional sets.

3. The subcontractor is not sure of his learning curve. The prime contractor's experience for new-type production, however, indicates approximately an 81-percent learning curve, which is compatible with the industry. This learning curve, therefore, is being used to evaluate his quotation.

4. The subcontractor recently had agreed to an 8-percent wage increase. His average wage rate estimated in the quotation for the first 50 sets was $1.80 per direct man-hour.

5. The subcontractor's total overhead rate is approximately 150 percent of his direct labor costs.

6. The subcontractor is anticipating a 10-percent profit on estimated cost for the repeat order but estimates that only a 3 percent profit on cost will be realized for the initial order of 50 sets.

Based on this information, computations would be as follows:

1. Original price per set for 50 sets minus the following items:		$1,800.00
Profit at 3 percent on cost	$ 52.43	
Tooling: $12,000 ÷ 50 sets	$240.00	
Material cost per set	$120.00	
		$ 412.43
Cost of direct labor plus overhead per set		$1,387.57

2. Estimated direct man-hour dollars per set in initial 50 sets is determined at $1,387.57 ÷ 2.5 or $555.03 (1 unit for direct man-hours and 1.5 units for overhead).

3. Anticipated direct man-hours required per set for 50 sets = $555.03 ÷ $1.80 per direct man-hour, or 308.4 direct man-hours.

4. Man-hours for unit 1 = $308.4 \times 50^{0.3040}$ = 1,013 direct man-hours.

5. Average man-hours for 350 sets = $\frac{1,013}{350^{0.3040}}$ = 170.7 direct man-hours.

6. Estimated average direct man-hours for 300 additional sets will be: $\frac{350 \times 170.7 - 50 \times 308.4}{300}$ = 147.75, rounded to 148 direct man-hours per set.

7. Estimated price breakdown per set (excluding any additional tooling requirements) for 300 additional sets.

a.	Direct material cost of $120 per set plus 4-percent increase	$124.80
b.	Direct man-hours per set, 148	
c.	Direct man-hour dollars ($1.80 per hour plus 8 percent) = $1.94	$287.12
d.	Overhead of 150 percent of direct man-hours	$430.68
	Total estimated cost per set	$842.60
e.	Profit of 10 percent of cost	$ 84.26
	Estimated selling price per set (for 300 additional sets)	$926.86

APPENDIX B

SOLUTION OF PROBLEMS IN WHICH THE UNIT TIME CURVE IS A STRAIGHT LINE ON LOG-LOG GRAPH PAPER OF THE TYPE $y = \frac{K}{x^n}$

I. Problem Number One.

A. Problem.

Find the estimated direct man-hours required to produce the 185th unit using 750 direct man-hours for the first unit and an 80-percent learning curve.

B. Solution.

Direct man-hours for the 185th unit $= \frac{750}{185^{0.3219}} = 402$ direct man-hours.

II. Problem Number Two.

A. Problem.

Find the total estimated direct man-hours for 185 units, using 750 direct man-hours for the first unit and an 80-percent learning curve.

B. Solution.

$$\text{Total direct man-hours} = \frac{750}{1 - 0.3219}\left[(185 + 0.5)^{1 - 0.3219} - (0.5)^{1 - 0.3219}\right]$$

$$= \frac{750}{0.6781}\left[(185.5)^{0.6781} - (0.5)^{0.6781}\right]$$

$$= 37{,}500 \text{ direct man-hours.}$$

III. Problem Number Three.

A. Problem.

Find the average direct man-hours per unit for 185 units, using 750 direct man-hours for the first unit and an 80-percent learning curve.

B. Solution.

The average direct man-hours per unit for a given number of units can be found by dividing the total direct man-hours for the corresponding number of units by the number of units. The total direct man-hours for 185 units was found to be 37,500 direct man-hours in problem 2.

$$\text{Average direct man-hours per unit} = \frac{37,500}{185} = 203 \text{ direct man-hours.}$$

IV. Problem Number Four.

A. Problem.

Find the average direct man-hours per unit for a block of units 185 through 235, using 750 direct man-hours for the first unit and an 80-percent learning curve.

B. Solution.

1. Find the total direct man-hours for a block of units 185 through 235.

$$\text{Total direct man-hours} = \frac{750}{1 - 0.3219} \left[(235 + 0.5)^{1 - 0.3219} - (185 - 0.5)^{1 - 0.3219} \right]$$

$$= \frac{750}{0.6781} \left[(235.5)^{0.6781} - (184.5)^{0.6781} \right]$$

$$= 6,860 \text{ direct man-hours}$$

2. Average direct man-hours per unit $= \dfrac{6,860}{235 - 185} = \dfrac{6,860}{50} = 137$ direct man-hours.